Electrical Cirluits (2)

Section (3)

1/3/2015 → 3/3/2015

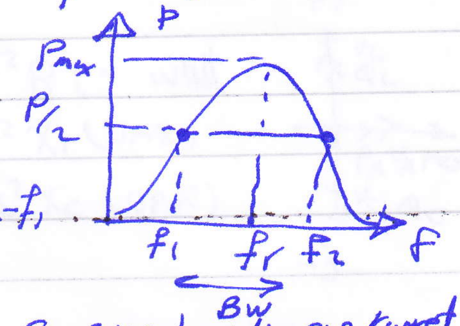
مطبعة جامعة بنها

أهم نقاط المحاضرة

[1] What is Resonance

→ Frequency selectivity in Radio and TV Communication Circuit.

→ Resonance Circuit pass range of Freq. & Reject other.

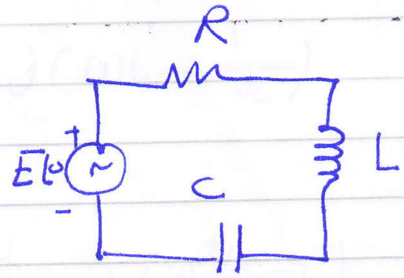


→ Filter Responce

- → f_r Resonance Freq. (center)
- → BW Bandwidth (passband) $= f_2 - f_1$

→ Resonance Circuit Consists of L, C, supply voltage/current.

[2] Series Resonance



Total impedance (Z_t)

$$Z_t = R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

→ Condition of series Resonance $X_L = X_C$ or $Z_t = R$

$\therefore \omega L = \frac{1}{\omega C}$ or $\omega^2 = \frac{1}{LC}$

$\therefore \omega_o = \frac{1}{\sqrt{LC}}$ rad/s or $f_o = \frac{1}{2\pi\sqrt{LC}}$ Hz

3 Analysis of RLC series circuit

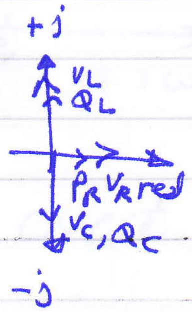
$Z_t = R$ (at resonance only)

$I = \frac{E\angle 0}{Z} = \frac{E}{R}\angle 0$

$V_R = I R \angle 0$ ⟷ $P_r = I^2 R \angle 0$ watt

$V_L = I X_L \angle 90$ ⟷ $Q_L = I^2 X_L$ (VAR)

$V_C = I X_C \angle -90$ ⟷ $Q_C = I^2 X_C$ (VAR)



4 impedance of Series RLC versus frequency.

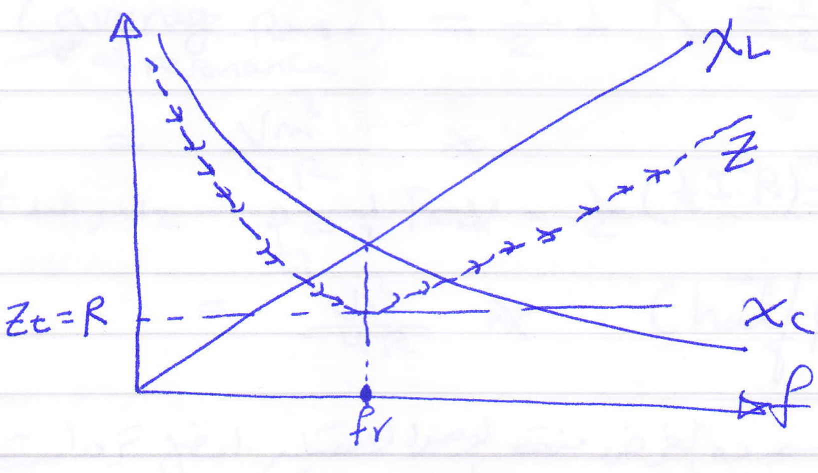
$Z = R + j\omega L - j\frac{1}{\omega C} = R + j(\omega L - \frac{1}{\omega C})$

$= R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$

$= \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2} \angle \tan^{-1}\left(\frac{\omega^2 LC - 1}{\omega RC}\right)$

(Magnitude) (→ phase)

Note at $\omega = \omega_s \rightarrow Z_t = R$, $\theta = 0$



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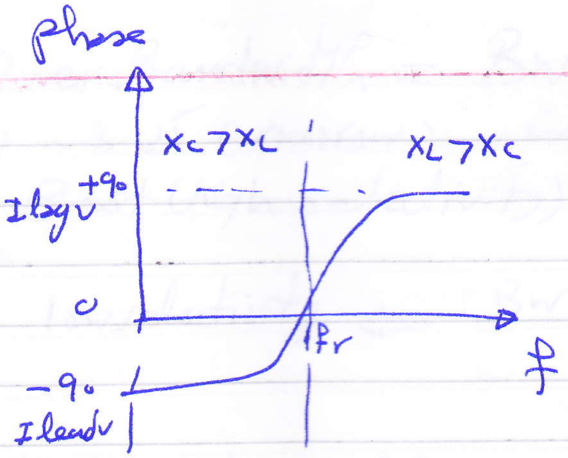
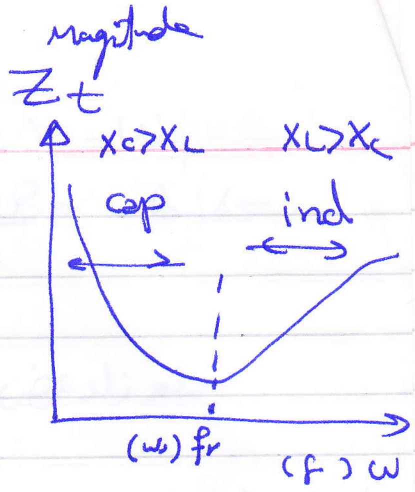




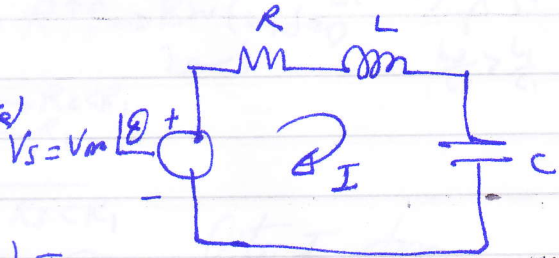
5 Current and Power in series Resonance Circuit

→ (resonance)

$$I = \frac{V}{Z} = \frac{V_m \angle \theta}{R}$$

↳ $Z = R$ (resonance)

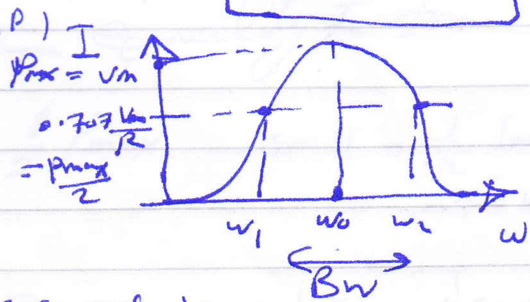
$I_{max} = V_m / R$



→ (not resonance)

$$I = |I| = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

↓ magnitude



عند ال resonance ← اقصى قيمة للتيار $\frac{V_m}{R}$

وعند قيمة تردد (freq.) (ω₀) ← يقل التيار

→ Power (average power) at resonance $= \frac{1}{2} I^2 R = \frac{1}{2} \left(\frac{V_m}{R}\right)^2 R$

$$= \frac{V_m^2}{2R} \quad *$$

→ at ω_1, ω_2: $P_{\omega_1, \omega_2} = \frac{1}{2} P_{max} = \frac{1}{2}\left(\frac{1}{2} I^2 R\right) = \frac{1}{4} I^2 R$

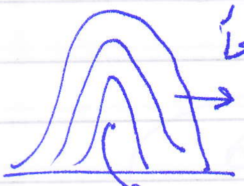
$$= \frac{V_m^2}{4R} \quad *$$

(half power freq.)

والرسم ده الغرض منه الوصول للاختيار، افضل ترددات للمحطات للإذاعة/TV

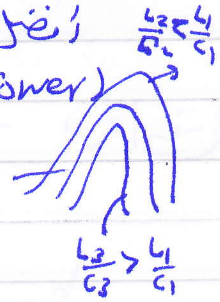
6 half power Bandwidth $= BW = \omega_2 - \omega_1$

كلما BW صغر (narrow) كلما أعلى
= Best (high selectivity)

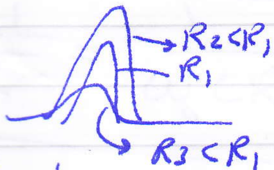


ولو زاد عرض BW ⟸ low selectivity

<u>Notes</u> **1** If $\frac{L}{C}$ ↑↑ → BW (narrower)



2 If L, C const $\therefore$ R ↑↑ → BW (widen) يوسع أوسع



7 Bandwidth selectivity — quality factor

at half power point $P = \frac{1}{2} P_{max}$ or $I = \frac{1}{\sqrt{2}} I_{max} = 0.707 \frac{V_m}{R}$

$$Z = \sqrt{2}\, R$$

$\therefore \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}\, R$ لكي نحصل على ω

$$I = \frac{1}{\sqrt{2}} \frac{V_m}{R}$$
$$\frac{V}{Z} = \frac{1}{\sqrt{2}} \frac{V_m}{R}$$
$$\therefore Z = \sqrt{2}\, R$$

$$\therefore \omega_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

$$\omega_2 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$BW = \omega_2 - \omega_1 = \frac{R}{L} \quad \text{rad/s}$$

$$\omega_0 = \sqrt{\omega_1 \omega_2} \quad \text{resonant freq.}$$

(5)

→ sharpness of response in Resonance measured by Q

$$Q \text{ (quality Factor)} = 2\pi \frac{\text{Peak energy stored in circuit}}{\text{Energy dissipated by ckt}}$$

$$Q = \frac{\text{Reactive power}}{\text{average power}}$$ ليس هاوءات

$$Q = \frac{I^2 X_L}{I^2 R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

OR $$Q = 2\pi \frac{\frac{1}{2} L I^2}{\frac{1}{2} I^2 R (1/f)} = \frac{2\pi f L}{R}$$ طريقة أخرى

في المنحنى

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$ → at Resonance

$$Q = \frac{\omega L}{R} = \frac{1}{\omega R C}$$ → (1)

$$B = \frac{R}{L} = \frac{\omega_0}{Q} = \omega^2 C R$$ → (2)

(B: BW)

$$Q = \omega_0 / B$$ → (3)

Q ↑↑ → more selectivity → smaller BW

اذا كان Q ≫ 10

(4) $\omega_1 \simeq \omega_0 - B/2$

$\omega_2 \simeq \omega_0 + B/2$

end of lecture

ملخص القوانين

[1] $\omega_0 = \frac{1}{\sqrt{LC}}$, $f_0 = \frac{1}{2\pi\sqrt{LC}}$

[2] $Z = R + j(\omega L - \frac{1}{\omega C})$ → total series Res.

$Z = R$ (at resonance).

[3] $BW = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0}{Q}$

[4] $\omega_{1\,actual} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_0 - B/2$ → from half power

$\omega_{2\,actual} = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} \cong \omega_0 + B/2$ → from half power

} at $Z = \sqrt{2}R$

[5] $\omega_0 = \sqrt{\omega_1 \omega_2}$

[6] $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

Sheet (2)

1 A series RLC has $R = 2K$, $L = 40mH$, $C = 1\mu F$. Calc. the Impedance at Resonance and at one Fourth, one half, twice, Four times resonant freq.

Sol

(1) أولاً نجيب قيمة ω

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(40 \times 10^{-3})(1 \times 10^{-6})}} = 5000 \text{ rad/s} = 5 \text{ Krad/s}$$

$$* \quad Z = R + j\omega L - j\left(\frac{1}{\omega C}\right) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

a- For Resonance $Z = R_p = 2K$

b- " one Fourth $\left(\omega_b = \frac{1}{4}\omega = \frac{5}{4} \text{ krad/s}\right)$

$$\therefore Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = 2000 + j\left(\frac{5}{4} \times 40 \times 10^{-3} - \frac{1}{\frac{5}{4} \times 1 \times 10^{-6}}\right)$$

$$= (2 - j0.75) \, k\Omega$$

c- one half $\omega_c = \frac{1}{2}\omega \Rightarrow Z = 2 - j0.3 \, k\Omega$

d- Twice $\omega_d = 2\omega \rightarrow Z = 2 + j0.3 \, k\Omega$

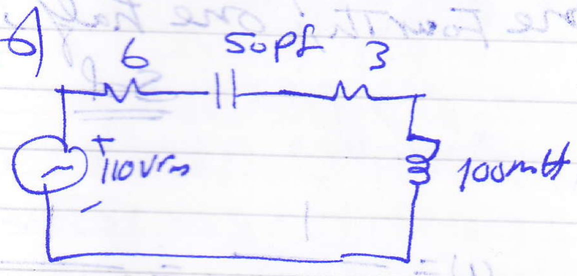
e- Four times $\omega_e = 4\omega \rightarrow Z = 2 + j0.75 \, k\Omega$

8

[2] Coil with resistance 3Ω, inductance 100mH, connected in series with capacitor 50pF, a resistor of 6Ω and signal generator 110 Vrms; calc. ω_0, Q, B at Resonance.

Sol//

Sol



$R_t = 6+3 = 9$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{(100\times10^{-3})(50\times10^{-12})}} = 447.21$ krad/s

$\rightarrow Q = \frac{\omega L}{R} = \frac{(447.21)\times10^{3} \times 100\times10^{-3}}{9} = 4969$

$\Rightarrow B = \omega/Q = \frac{447.21\times10^{3}}{4969} = 90$ rad/s

or $= R/L = 9/100\,mH = 90$ rad/s

[3] Design RLC circuit with $B = 20$ rad/s, $\omega_0 = 1000$ rad/s, find Q

Sol// Let $R = 100\Omega$

$\therefore B = R/L \Rightarrow$ ~~L =~~ $R/B = 100/20 = 5\,H$

$\omega = \frac{1}{\sqrt{LC}}$ 6 $\omega^2 = \frac{1}{LC}$ 6 $C = \frac{1}{\omega^2 L} = \frac{1}{(1000)^2(5)}$

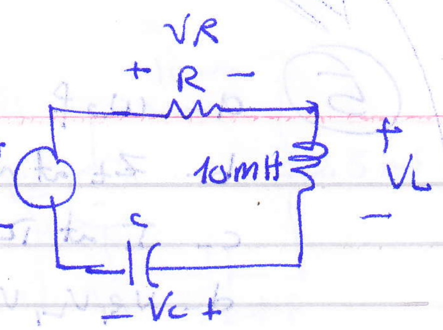
$= 20\,\mu F$

$Q = \frac{\omega L}{R} = \frac{(1000)(\cancel{100})}{100} = 50$

9 | 10

4

a- Determine R, C (at resonance) freq ($f_r = 25kHz$, rms and 25mA)



b- power dissipated by circuit at resonance

c- Phasor voltage Vc, VL, VR at resonance

d- write sinusoidal eq. for Vc, VL, VR.

80

a- $F_r = 25kHz$, $I_{rms} = 25mA$, $V_{rms} = 625mV$

∴ $Z = R$ at resonance

∴ $Z = \frac{V}{I} = \frac{625}{25} = \boxed{25 = R}$

for C: $f_r = \frac{1}{2\pi\sqrt{LC}} = 25\times10^3 Hz$

∴ $\frac{1}{\sqrt{LC}} = 25\times10^3 \times 2\times\pi$

∴ $\frac{1}{LC} = (25\times10^3 \times 2\pi)^2$

∴ $\frac{1}{(10\times10^{-3})C} = (25\times10^3\times2\pi)^2$

∴ $C = \frac{1}{(10\times10^{-3})(25\times10^3\times2\pi)^2} = \boxed{4.05\times10^{-9} F}$ nF

b- $P = VI = (625mV)(25mA) = 15.6mW$

c- $V_R = IR\angle 0 = (25mA)(25) = 0.625V\angle 0$

$V_L = IX_L = (25mA)(2\pi f L)\angle 90 = (25mA)(2\pi\times25\times10^3\times10\times10^{-3})\angle{+90}$ (X_L → ωL)

$= 39.25\angle 90$

$V_C = IX_C = (25mA)\left(\frac{1}{2\pi f c}\right)\angle{-90} = (25mA)\left(\frac{1}{2\pi\times25\times10^3\times4.05\times10^{-9}}\right)\angle{-90}$

$= 39.31\angle{-90}$

~~$V_C = V_m \sin(\omega t + \theta) = 39.31 \sin$~~

10

5

a- ω_0, f

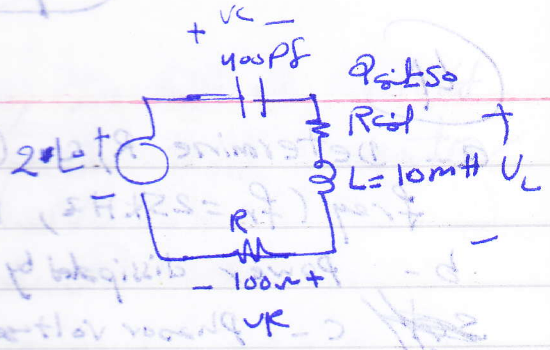
b- Z_t at resonance

c- I at Res.

d- V_R, V_L, V_C at Res.

e- P dissipated (Q_C, Q_L) Power

f = Quality factor at Res



Sol

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(400\times10^{-12})(10\times10^{-3})}} = 500\ krad/s$

$f = \frac{\omega}{2\pi} = 79.617\ KHZ$

b- $Z_t|_{res} = R + R_{coil} = 100 + R_{coil}$

$\because Q_{coil} = \frac{\omega L}{R_{coil}} \therefore R_{coil} = \frac{\omega L}{Q} = \frac{500\times10^{3}\times10\times10^{-3}}{50} = 100$

$\therefore Z_t = R = 100 + 100 = 200\angle 0$

c- $I_{res} = \frac{V}{Z_{t\,res}} = \frac{V}{R_t} = \frac{2}{200} = 0.01A\angle 0 = 10mA\angle 0$

d- $V_R = IR\angle 0 = (0.01)(100) = 1\angle 0$

$V_C = IX_C = (0.01)(\frac{1}{\omega C})\angle -90 = 0.01 \frac{1}{(500)(10^3)(400)(10^{-12})}\angle -90 = 50\angle -90$

$V_L = I(X_L + R) = (0.01)(R + jX_L)$

$= (0.01)(100 + j(500\times10^{3})(10\times10^{-3})) = (0.01)(100 + j5000)$

$= 1 + j50 = \sqrt{1^2 + 50^2}\angle \tan^{-1}(\frac{50}{1}) = 50.01\angle 88.85^\circ$

e- $P = VI = (2)(0.01) = 0.02 = 20mW$

$Q_C = I^2 X_C = (0.01)(50) = 0.5\ VAR$

$Q_L = I^2 X_L = (0.01)(50.01) = 0.501\ VAR$

$Q = \frac{\omega L}{R_t} = \frac{(500\times10^{3})(10\times10^{-3})}{200} = 25$

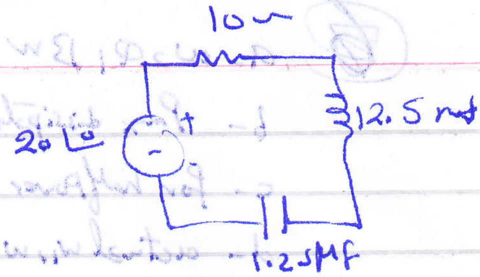
— 11 —

Assignmt

a- ω   b- Zt   Z∠θ → 0.1 ω, 0.2 ω, 0.5 ω, ω, 2 ω, 5 ω, 10 ω

c- draw |Z| ⟷ ω
θ ⟷ ω

d- |I| = ?!   e- plot |I| ⟷ ω



Sol

a- $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(12.5\times10^{-3})(1.25\times10^{-6})}} = 8000$ rad/s

b- $Z_{t\,res}(\omega) = R = 10\angle 0$

$Z|_{0.1\omega} = R + j(\omega L - \frac{1}{\omega C}) = 10 + j(\frac{100}{10} - \frac{100\times 0.1}{0.01}) = 10 + j(10 - 1000)$

$= 10 - j990 = 990\angle -89.42$

$Z|_{0.2} = 10 + j(100\times 0.2 - 100/0.2) = 10 - 480j = 480.1\angle -88.8$

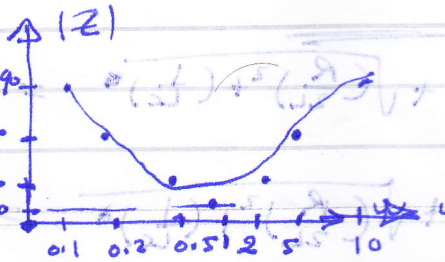
$Z|_{0.5\omega} = 10 + j(100\times 0.5 - 100/0.5) = 10 - j150 = 150.3\angle -86.18$

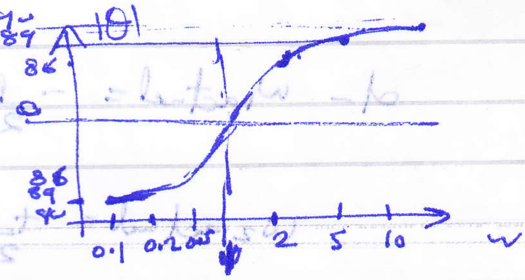
$Z|_{2\omega} = 10 + j(100\times 2 - 100/2) = 10 + j150 = 150.3\angle 86.18$

$Z|_{10\omega} = 10 + j(100\times 10 - 100/10) = 10 + j990 = 990\angle 89.42$

$Z|_{5\omega} = 10 + j(100\times 5 - 100/5) = 10 + j480 = 480.1\angle 88.8$

c-





d- $I|_{\omega} = V/R = \frac{20}{10} = 2\angle 0$

$I|_{0.1\omega} = V/Z = \frac{20}{990\angle -89.4} = 0.02\angle 89.4$

$I|_{0.2\omega} = 0.042\angle 88.8 = \frac{20}{480}$

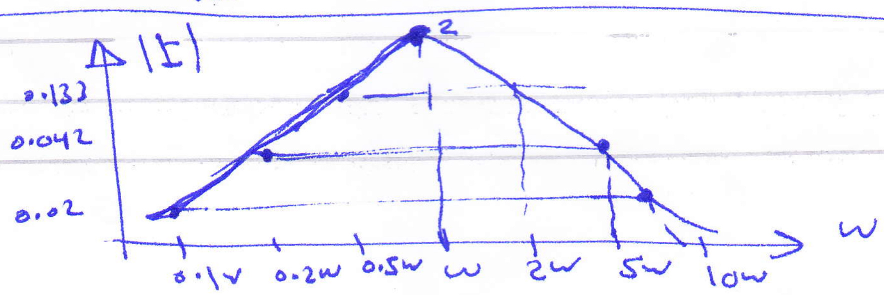
$I|_{0.5\omega} = \frac{20}{150} = 0.133\angle 86.18$

$I|_{2\omega} = \frac{20}{150\angle} = 0.133\angle -86.18$

$I|_{5\omega} = \frac{20}{480\angle} = 0.042\angle -88.8$

$I|_{10\omega} = \frac{20}{990} = 0.02\angle -89.8$

e-

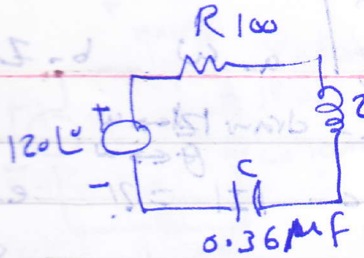


(6) a- ω, Q, BW

b- P_{max} dissipated

c- for half power freq. find ω_1, ω_2

d- actual ω_1, ω_2



Sol

a- $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(200\times10^{-3})(0.36\times10^{-6})}} = 3726$

$Q = \frac{\omega L}{R} = \frac{3726 \times 200\times10^{-3}}{100} = 7.452$

$BW = \omega / Q = 3726 / 7.452 = 500$ rad/sec

b- $P_{max} = V_m I_m = V_m\left(\frac{V_m}{R}\right) = V_m^2/R = \frac{(120)^2}{100} = 144$ W

Max [illegible] rms [illegible]

c- $\omega_1 = \omega_0 - B/2 = 3726 - \frac{500}{2} = 3476$ rad/s

$\omega_2 = \omega_0 + B/2 = 3726 + \frac{500}{2} = 3976$ rad/s

d- $\omega_{1\,actual} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = 3485.16$ rad/s

$\omega_{2\,actual} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} = 3985$ rad/s

End of sheet 2